Roll Number		

SET - 1

INDIAN SCHOOL MUSCAT
FIRST TERM EXAMINATION

MATHEMATICS

CLASS: XII | Sub. Code: 041 | Time Allotted: 3 Hrs

02.05.2018 | Max. Marks: 100

GENERAL INSTRUCTIONS:

1. The question paper consists of 29 questions.
2. Q(1 - 4) are of 1 mark each , Q(5 – 12) are of 2 marks each, Q(13 – 23) are of 4 marks each and Q(24-29) are of 6 marks each.
3. All questions are compulsory however wherever indicated for internal option , do any one out of the two.

SECTION A

1. Let $* : R \times R \to R$ given by $(a, b) \to a + 4b^2$ is a binary operation. Compute $(-5) * (2 * 0)$.

2. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

3. Solve for x : $\tan^{-1} x + \sin^{-1}(-1) + \cot^{-1}\frac{3}{4} = 0$

4. For what value of 'k' is the function $f(x) = \begin{cases} \frac{\sin 5x}{3x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ continuous at x = 0 ?

SECTION B

5. Show that the binary operation * on $A = \mathbf{R} - \{-1\}$ defined as $a*b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A.

6. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = x(x + 1)^{-1}$ then show that the function is one one.

7. Find the numerical value of $\tan\{ 2(\tan^{-1}(0.2)) - 45^\circ \}$

8. **Discuss the** differentiability **of the function** $f(x) = |x-2|$ at $x = 2$.

9 Find the derivative of $\sin\left(\cos^2(\sqrt{x})\right)$.

10 Find the value of derivative of $(x-1)^{\frac{2}{3}}(x+1)^{\frac{1}{3}}$ at $x = 0$

11 If $f(x) = \log x$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, find $\frac{dy}{dx}$

12 The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

SECTION C

13 If A = { 1, 2, 3} and R = {(1,2), (1,1),(2,3)} be a relation on A . What minimum number of elements may be adjoined with the elements of R so that R may become transitive as well as symmetric but not reflexive. List those elements of R and justify your answer.

14 Let S = {a, b, c} and T = {1, 2, 3} and f and g are two functions defined from S to T as follows f = {(a,3), (b,2), (c,1)} and g = {(a,1), (b,2), (c, 1))}
a) Check if f and g are one one and onto? Justify.
b) List the elements of $f^{-1}$and g^{-1} if they exist.

15 Prove that : $\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$

OR

Solve the equation for x : $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

16 Prove the following : $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

17 Find the value of :

$$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right] + \sin\left[\frac{\pi}{2} - \sin^{-1}\frac{\sqrt{3}}{2}\right]$$

18 Find whether the following function is differentiable at x = 1 and x = 2 or not :

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

19 Verify Mean Value theorem for the function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$.

OR

Discuss the applicability of Rolle's theorem for the following function on the indicated interval :

$f(x) = 3 + (x-2)^{2/3}$ on $[1, 3]$

20 If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$.

OR

If $(x-y) \cdot e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$.

21 Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

22 Find the local maximum and local minimum of $f(x) = 2\sin x - x$, $x \in [0, 2\pi]$. Also find the maximum and minimum values of f(x) at these points.

23 The length x of a rectangle is decreasing at the rate of 5 cm/sec and the width y is increasing as the rate of 4 cm/sec when x = 8 cm and y = 6 cm. Find the rate of change of

(a) Perimeter (b) Area of the rectangle.

SECTION D

24 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

25 If f, g : R → R be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, $\forall x \in R$. Then find fog and gof. Hence find fog(–3), fog(5) and gof (–2).

OR

Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

26 If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

27 Find the derivative of $\sqrt{4 + \sqrt{4 + \sqrt{4 + x^2}}}$

OR

Differentiate wrt x : $\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$

28 The sum of the surface areas of a cuboid with sides x, $2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.

29 Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$.

OR

Find the intervals in which the function $f(x) = \dfrac{4\sin x}{2+\cos x} - x; 0 \leqslant x \leqslant 2$ strictly increasing or strictly decreasing.

End of the Question Paper